AVERAGE AND PROBABILITY,

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

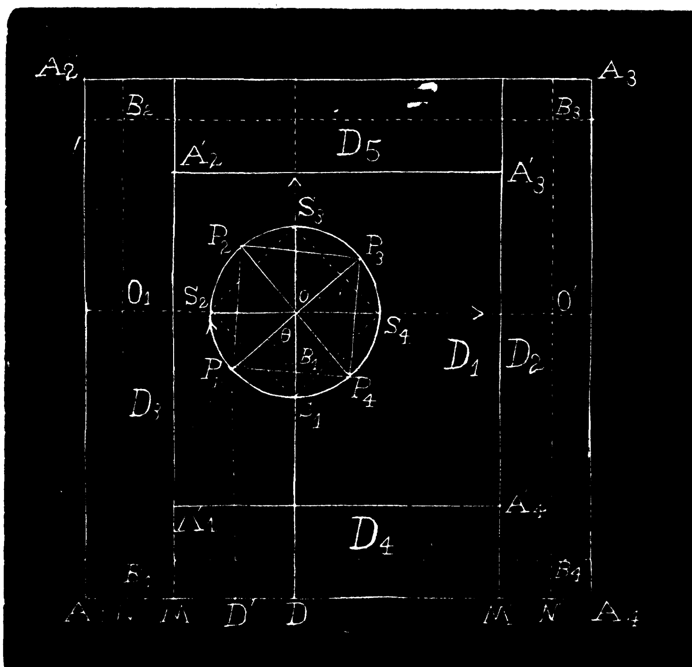
SOLUTIONS OF PROBLEMS.

20. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

A surface one inch square is thrown at random upon a surface one foot square, but in such a manner as always to lie wholly upon the larger surface. Find the mean value of *the sum of the distances* of the vertices of the smaller surface, from *any* vertex of the larger surface.

Solution by the PROPOSER.

Let $S_1S_2=s$, $A_1A_2=12s$, $AD=x$, $DO=y$; then $S_1O=\frac{1}{2}s\sqrt{2}$. Put $\angle S_1OP_1=\angle S_2OP_2=\angle S_3OP_3=\angle S_4OP_4=\theta$; then $DD'=\frac{1}{2}s\sqrt{2}\sin\theta$, and $(DO-P_1D')=\frac{1}{2}s\sqrt{2}\cos\theta$.



$\therefore$ $A_1D'=\frac{1}{2}(2x-s\sqrt{2}\sin\theta)$, and $P_1D'=\frac{1}{2}(2y-s\sqrt{2}\cos\theta)$; also, $\Delta_1=A_1P_1=\frac{1}{2}\sqrt{[(2x-s\sqrt{2}\sin\theta)^2+(2y-s\sqrt{2}\cos\theta)^2]}$, $\Delta_2=A_1P_2=\frac{1}{2}\sqrt{[(2x-s\sqrt{2}\cos\theta)^2+(2y+s\sqrt{2}\sin\theta)^2]}$, $\Delta_3=A_1P_3=\frac{1}{2}\sqrt{[(2x+s\sqrt{2}\sin\theta)^2+(2y+s\sqrt{2}\cos\theta)^2]}$, $\Delta_4=A_1P_4=\frac{1}{2}\sqrt{[(2x+s\sqrt{2}\cos\theta)^2+(2y-s\sqrt{2}\sin\theta)^2]}$.

Let $\Delta=(\Delta_1+\Delta_2+\Delta_3+\Delta_4)$; then five cases confront us, for consideration.

First Case—The point O may lie in the square surface $A'_1A'_2A'_3A'_4$. If $\frac{1}{8}(24-\surd 2)s=a$ and $\frac{1}{8}s\surd 2=b$, the mean value of the sum of the distances in this case becomes

$$\mathbf{D}_1=\int_b^a\int_b^a\int_0^{\frac{1}{2}\pi}[\triangle_1+\triangle_2+\triangle_3+\triangle_4]dxdyd\theta\div\int_b^a\int_b^a\int_0^{\frac{1}{2}\pi}dxdyd\theta\ldots.(1).$$

Second Case—The point O may lie in the rectangular surface D_2; then if $11\frac{1}{8}s=c$, $\frac{1}{8}s=e$, and $\sin^{-1}(x\,/\,e\surd 2)=\phi$, the mean value in this case becomes

$$\mathbf{D}_2=\int_a^c\int_e^c\int_0^\phi[\triangle_1+\triangle_2+\triangle_3+\triangle_4]dxdyd\theta\div\int_a^c\int_e^c\int_0^\phi dxdyd\theta\ldots.(2).$$

Third Case—The point O may lie in the rectangular surface D_3; and in this case,

$$\mathbf{D}_3=\int_e^b\int_e^c\int_0^\phi[\triangle_1+\triangle_2+\triangle_3+\triangle_4]dxdyd\theta\div\int_e^b\int_e^c\int_0^\phi dxdyd\theta\ldots.(3).$$

Fourth Case—The point O may lie in the rectangular surface D_4.Put $\frac{1}{8}s(23-\surd 2)=f$, and $\frac{1}{8}s(\surd 2+1)=g$; then the mean value in this case becomes

$$\mathbf{D}_4=\int_g^f\int_e^b\int_0^\phi[\triangle_1+\triangle_2+\triangle_3+\triangle_4]dxdyd\theta\div\int_g^f\int_e^b\int_0^\phi dxdyd\theta\ldots.(4).$$

Fifth Case—The point O may lie in the rectangular surface D_5; then the mean value in this case becomes

$$\mathbf{D}_5=\int_g^f\int_a^c\int_0^\phi[\triangle_1+\triangle_2+\triangle_3+\triangle_4]dxdyd\theta\div\int_g^f\int_a^c\int_0^\phi dxdyd\theta\ldots.(5).$$

Consequently the *required* mean value becomes

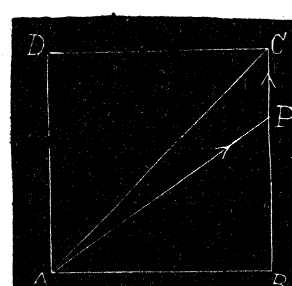
$$\mathbf{D}=\tfrac{1}{5}(\mathbf{D}_1+\mathbf{D}_2+\mathbf{D}_3+\mathbf{D}_4+\mathbf{D}_5)\ldots.(6);$$

and the labor required in the performance of the integrations indicated is simply enormous.

21. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

From one corner of a square field, a boy runs in a random direction, with a random uniform velocity. The greatest distance the boy can run in one minute is equal to the diagonal of the field. What is the probability that the boy will be in the field at the end of one minute?

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.



Let $AB=a$, and $BP=x$; then $AP=\surd(x^2+a^2)=w$, and $AC=a\surd 2=m$. The boy will be in the field at the expiration of $t=1$ minute, if v be not greater than $\surd(x^2+a^2)$ Hence the required probability becomes

$$\mathbf{P}=\int_0^a\int_0^w dxdv\div\int_0^a\int_0^m dxdv,=\frac{1}{a^2\surd 2}\int_0^a\surd(x^2+a^2)dx$$